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# **Implementation of Altimetry Data In the GIPSY POD Software Package**

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## **ABSTRACT**

Altimetry data has been used extensively to acquire data about characteristics of the Earth, the Moon and Mars. More recently, the idea of using altimetry for orbit determination has also been explored. This report discusses modifications to JPL's GIPSY/OASIS II software to include altimetry data as an observation type for precise orbit determination. The mathematical foundation of using altimetry for the purpose of orbit determination is presented, along with results.

## **INTRODUCTION**

Over the past few decades, altimetry has been commonly used to aid in geophysical research by directly measuring the distance between a space vehicle and the Earth's surface. The first radar altimeter to fly onboard a spacecraft was deployed by Skylab in 1973. Since then, the accuracy of altimeter measurements, as well as the error corrections for the measurements have been improved dramatically. Altimetry data have been used to better understand Earth's topography and sea surface height, as well as aid in the mapping of the lunar and Martian surfaces.

The GIPSY precise orbit determination (POD) software package previously did not support the use of altimetry data. Under JPL Contract 1218656, Colorado Center for Astrodynamics Research (CCAR) developed modifications to several modules that enable GIPSY to use altimetry data as an observation type in the orbit determination process. In order to include the altimetry data type, the equations for computing altimeter height were developed. The partials with respect to the state were then calculated. Unlike laser tracking, the location of the ground station is not known. Instead, the measurement is the distance from the altimeter to the sub-satellite position on the surface of the ocean. The height of the ocean at any given point must be modeled, and then the sub-satellite position can be determined. It is assumed throughout that the satellite is nadir pointing, and no attempt is made to incorporate attitude data in with the altimeter data. Further complicating the problem is the fact that the height of the ocean surface varies both spatially and temporally.

The following mathematical model development applies to sea surface altimetry data. In order to use altimetry data over land, the model would have to be modified to account for the Earth's topography.

## ALTIMETRY MATHEMATICAL MODEL

The altimeter observation is derived by measuring the time it takes a radar or laser pulse to travel from the altimeter to the ground (or sea surface) and back again. The distance the signal traveled is then simply the travel time,  $\Delta t$ , multiplied by the speed of light,  $c$ . Thus the height above the surface,  $h$ , is approximately given by equation (1).

$$h = c \frac{\Delta t}{2} \quad (1)$$

It will be assumed that the actual observation,  $\Delta t$ , will be converted to height,  $h$ , as a matter of convenience. The observation will not represent the actual height of the altimeter above the surface due to several error sources. First, various sensor specific corrections must be applied. It will be assumed that these corrections will be made during altimeter preprocessing. Assuming these corrections are known, a mathematical model for the altimeter height,  $h$ , is given by equation (2).

$$h = h_s - h_c + h_a + b + \eta \quad (2)$$

where:

$h_s$  = The geometric distance between the spacecraft center of mass and the point of pulse contact on the mathematical model of the surface.

$h_c$  = The correction due to the fact that the electronic reference point does not coincide with the spacecraft center of mass.

$h_a$  = The correction due to atmospheric delay.

$b$  = The correction for a possible bias in  $h$ .

$\eta$  = The contribution of both systematic and random errors associated with the mathematical model. The errors associated with the sea surface model are discussed in more detail in the next section.

The geometric height,  $h_s$ , can be computed using equation (3)

$$h_s = [(\bar{r} - \bar{r}_s) \cdot (\bar{r} - \bar{r}_s)]^{1/2} \quad (3)$$

where

$\bar{r} = \bar{r}(x,y,z)$  = The geocentric ECF position of the spacecraft center of mass.

$\bar{r}_s = \bar{r}_s(x_s,y_s,z_s)$  = the geocentric ECF position of the point of pulse contact on the mathematical model of the surface.

The mathematical model of the surface must take into account static and dynamic aspects of the sea surface. The next section presents more detail on the sea surface model.

## SEA SURFACE MODEL

The static component of the sea surface height can be modeled using two distinct methods. The least complicated method is to use independent data to correct the altimeter height to the reference ellipsoid. In this case, the height of the satellite above the reference ellipsoid,  $h_e$ , is the observable. This distance is given by equation (4).

$$h_e = h + h_g \quad (4)$$

where  $h_g$  is the sea surface height above or below the reference ellipsoid (obtained using an independent sea surface model). It is assumed that  $h_g$  includes the corrections due to the mean sea surface, tidal and loading effects, atmospheric delays and instrument errors. The largest deviation of the sea surface from the reference ellipsoid is on the order of 100 meters (Rapp et al.), thus it is important to include  $h_g$  in the calculations. Given that the distance to the reference ellipsoid is considered, the reference surface,  $W$ , takes the form of equation (5).

$$W(x, y, z) = x^2 + y^2 + \frac{z^2}{1 - e^2} = R_e^2 \quad (5)$$

where,  $x$ ,  $y$ , and  $z$  are the sub-satellite geocentric coordinates,  $e$  is the eccentricity of the reference ellipsoid, and  $R_e$  is the ellipsoidal semi-major axis. This model simplifies the problem considerably. There are two main drawbacks to this method. First, the partial derivatives of  $h$  must be computed using the sub-satellite point on the reference ellipsoid. This will create a slight error due to the difference in the slope of the actual mean sea surface and the reference ellipse. Second, this method cannot be used to estimate the geopotential coefficients (and thus the mean sea surface) or the tides. To get around these problems, the sea surface can be approximated as an equipotential surface. This method is much more complicated and given the goals and resources of the current effort, the method was not considered.

## ALTIMETRY COMPUTATION

The altimeter observation residual is needed for the least squares orbit determination problem. The residual,  $\Delta h$ , is given by equation (6).

$$\Delta h = H - h \quad (6)$$

where:

$H$  = Observed altimetry height.

$h$  = Computed altimetry height.

Consider a spacecraft with position,  $\bar{r}(x, y, z)$ , and sub-satellite position,  $\bar{r}_s(x_s, y_s, z_s)$ . The altimeter measurement vector is given by equation (7).

$$\bar{h} = \bar{r} - \bar{r}_s \quad (7)$$

Since the altimeter height is being corrected to the reference ellipsoid,  $\bar{r}_s$  is the sub-satellite point on the reference ellipsoid. The altimeter height vector,  $\bar{h}$ , is assumed to be oriented perpendicular to the reference ellipse, thus the geodetic latitude,  $\phi_{gd}$ , and longitude,  $\lambda$ , of the satellite and sub-satellite point are identical. Using the method described in Vallado (pg. 25), the coordinates of the sub-satellite point are given by equation (8)

$$\bar{r}_s = \begin{bmatrix} x_s \\ y_s \\ z_s \end{bmatrix} = \begin{bmatrix} C_{\oplus} \cos(\phi_{gd}) \cos(\lambda) \\ C_{\oplus} \cos(\phi_{gd}) \sin(\lambda) \\ S_{\oplus} \sin(\phi_{gd}) \end{bmatrix} \quad (8)$$

where

$$C_{\oplus} = \frac{R_e}{\sqrt{1 - e^2 \sin^2(\phi_{gd})}}$$

$$S_{\oplus} = \frac{R_e(1 - e^2)}{\sqrt{1 - e^2 \sin^2(\phi_{gd})}}$$

The geodetic latitude and longitude are found using the coordinates of the spacecraft position. The procedure is given in Vallado (pp. 202-205). The longitude is given by equation (9).

$$\lambda = \tan^{-1} \left( \frac{y}{x} \right) \quad (9)$$

Computing the geodetic latitude requires an iterative scheme. The geocentric latitude, given by equation (10), is used as an initial guess.

$$\phi_{gd\_old} = \phi_{gc} = \sin^{-1} \left( \frac{z}{r} \right) \quad (10)$$

Then, equations (11) and (12) are iterated until  $\phi_{gd}$  converges to some tolerance (i.e.  $\phi_{gd\_new} - \phi_{gd\_old} < \text{tolerance}$ ).

$$C_{\oplus} = \frac{R_e}{\sqrt{1 - e^2 \sin^2(\phi_{gd\_old})}} \quad (11)$$

$$\phi_{gd\_new} = \tan^{-1} \left( \frac{z + C_{\oplus} e^2 \sin(\phi_{gd\_old})}{\sqrt{x^2 + y^2}} \right) \quad (12)$$

Once the sub-satellite coordinates are found, the altimetry height is easily computed using equation (7).

## ALTIMETRY PARTIAL DERIVATIVES

In order to use altimetry data in the orbit determination problem, the partials of the altimetry height with respect to the state must be calculated. As mentioned previously, for the purpose of computing the altimeter partial derivatives, the effects of the geoid undulations can be neglected. In order to obtain the partial derivative of  $h$ , it is assumed that  $\bar{r}$  and  $\bar{r}_s$  are collinear. This way,  $h$  is simply the magnitude of  $\bar{r}$  minus the magnitude of  $\bar{r}_s$ , equation (13). In general,  $\bar{r}$  and  $\bar{r}_s$  are not collinear, however the assumption is a good approximation when calculating the partial derivatives of  $h$ .

$$h = r - r_s \quad (13)$$

where,  $r_s$  is the distance between the center of the Earth and the sub-satellite position on the reference ellipsoid, which is given by equation (14).

$$r_s = a \left[ 1 - \left( f + \frac{3}{2} f^2 \right) \sin^2(\phi) + \frac{3}{2} f^2 \sin^4(\phi) \right] \quad (14)$$

where:

$a$  = Reference ellipsoid semi-major axis

$f$  = Reference ellipsoid flattening

$\phi$  = Geocentric latitude =  $z/r$

Plugging equation (14) into equation (13) and taking the partial derivative with respect to the spacecraft position yields equation (15).

$$\frac{\partial h}{\partial r_i} = \frac{r_i}{r} + \frac{1}{r} \left[ \left( 2af + 3af^2 \right) \left( \frac{z}{r} \right) - 6af^2 \left( \frac{z}{r} \right)^3 \right] \left[ \frac{\partial z}{\partial r_i} - \frac{zr_i}{r^2} \right] \quad (15)$$

where,  $r$  is the distance from the satellite to the center of the Earth and  $r_i$  represents the ECF coordinates of the satellite:  $x$ ,  $y$ , or  $z$ .

## ALTIMETER DATA USED FOR THE PROJECT

Altimeter data from Topex for the periods of 01 July 2000 to 10 July 2000 was used in this analysis. The data was provided by JPL in a text based file format, and were taken from the GDR records for Topex. The data has been corrected for various effects, and also contains sea surface from the GDR. This text file was then reformatted to be consistent with the output from a dump\_qm run, and the module dump2qm was used to create an altimeter data qm file. This qm file was then merged with the daily Topex GPS qm files for processing.

## MODIFICATIONS TO QREGRES

The altimeter model and partial derivatives routines were first created outside of GOA II, and coded in Fortran 77. These were then incorporated back into the qregres module using the Doris data model\_t.f routine as a guide. The altimeter data was treated as a range data type with no bias. Later processing required the inclusion of a range bias parameter in the solutions.

## INITIAL POD RESULTS

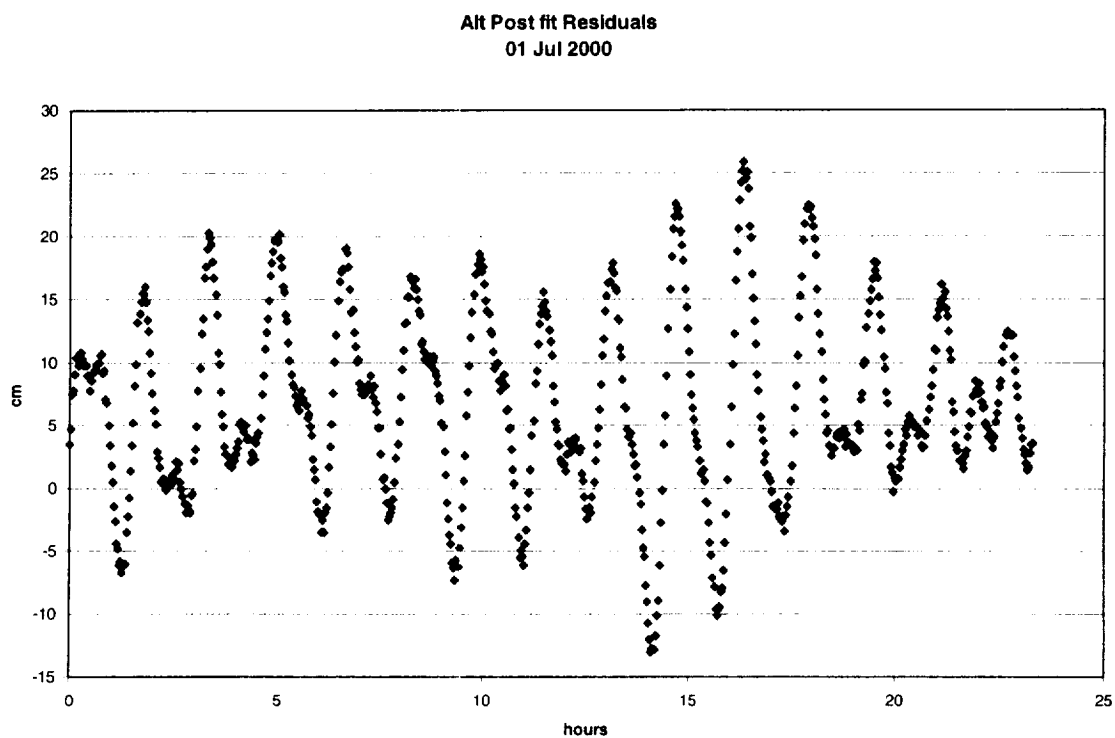
The OD process with the new data type began by first repeating the orbit generation process done with the JPL topex\_daily routines. Dynamic and reduced dynamic solution scenarios were used in this step which did not involve the use of altimeter data. Dynamic GPS orbits were generated until convergence, and then these were used to initialize a GPS+Altimeter data solution. The state estimated with the combined data type consisted of Drag, and spacecraft position and velocity. The once and twice per rev orbit parameters were held fixed to the values from the dynamic orbit solutions, and altimeter data noise was set to 30 cm. The resulting orbits differed slightly from the GPS only reduced dynamic solutions, which is to be expected with the relative weight of GPS phase data, as compared to altimeter data. When data noise was lowered to 10 cm, thus giving more weight to altimetry, the orbits degraded significantly. Statistics are shown below for a single day (01 July 2000), generated with the odiff module in a comparison between resulting orbits with GPS+Altimeter compared to GPS only reduced dynamic runs.

	Radial	Cross	Along	3D
Alt.				
Weight				
30 cm	1.0	0.3	1.2	1.6
10 cm	5.8	0.4	1.1	5.9

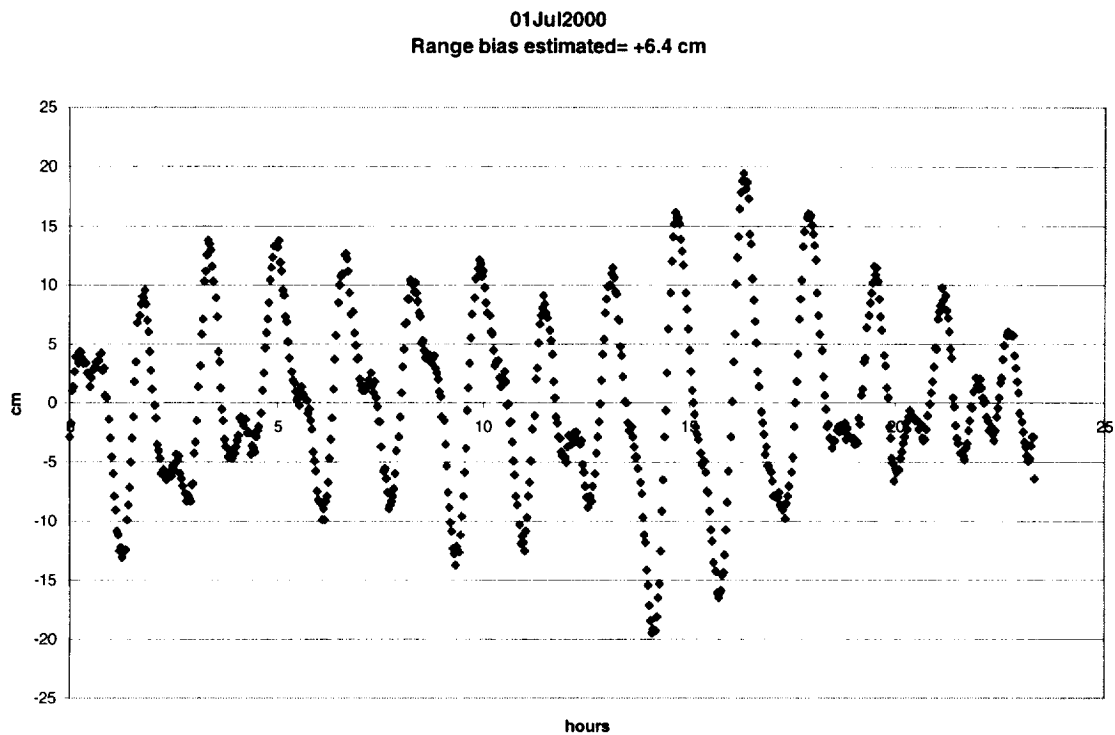
The altimeter data is much noisier than GPS phase, and the expected result was that this data caused the orbit solution to degrade.

## POSTFIT ALTIMETRY RESIDUALS

The postfit residuals for the altimetry data type show a pattern of once and twice per rev effects on the orbit. Orbit parameters for these periods are not estimated when the altimetry data is included, but are set to the final values from the converged dynamic runs. Initially, these residuals contained a bias of approximately 6 centimeters. When a range bias parameter is estimated for the altimetry data type, this bias drops to 3 mm.



**Figure 1 Altimeter post fit residuals**



**Figure 2 Altimeter postfit residuals, range bias parameter estimated**



The RMS of the residuals about the mean is 8.2 centimeters in both cases. The exact nature of the pattern is not understood, but could be consistent with either a 1 degree rotation in the sea surface height used in the analysis, or could be an ocean surface effect. Further study could involve mapping multiple days of orbit solutions onto a Lat/Lon grid to look for geographically correlated spatial effects.

## **SUMMARY OF RESULTS**

Modifications were completed to GIPSY-OASIS II to allow the simultaneous processing of altimeter range data along with satellite to satellite GPS data. Differences with reduced dynamic orbits show agreement to several centimeters, as expected. Post fit residuals for the altimetry data used in the process show correlated patterns of once and twice per rev signatures. The statistics of the residuals are consistent with what would be expected for this data type, however the patterns in the residuals are not completely understood. They are most likely due to errors in the ocean surface model, or perhaps due to a shift in the model from the GDR records.

## **FUTURE WORK**

If resources permit, an equipotential approximation of the sea surface height could be developed and implemented. Further investigations could also include dynamic sea surface topography model development to account for temporal changes in the sea surface height. If altimetry data with respect to land is required, a method to model the land topography needs to be developed. Another powerful application of altimeter data to POD is the utilization of altimeter crossover constraint equations. Using altimeter crossover methods such as those discussed by Rowlands et. al. (1999) and Shum et. al. (1990) could be especially useful for missions such as ICESat in which the spacecraft spends much of its time over land and ice. Crossover constraints over the ocean surface must include information about the dynamic ocean topography since the crossovers involve data from different points in time.

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